SCATTERING FROM 3-DIMENSIONAL DISCONTINUITIES IN MICROWAVE TRANSMISSION LINES

R.Sachs and F.J.Rosenbaum

Dept. of Electrical Engineering
Washington University
St. Louis, MO. 63130

ABSTRACT

A material body whose constitutive parameters are $\mu=\mu'-j\mu''$, $\epsilon=\epsilon'-j\epsilon''$ located in a strip line ,partially filling it,represents a 3D scattering problem. This problem is solved by the Reaction Method yielding the configuration's scattering parameters. An iterative procedure then enables the determination of μ,ϵ from the measured values of the S-parameters.

INTRODUCTION

Material bodies located in transmission lines will scatter incident waves if their constitutive parameters μ,ϵ differ from those of the surrounding medium. Some of the scattered energy is reflected to the source,some is transmitted past the discontinuity and some is stored in the higher order modes generated to match the boundary conditions. In most situations the discontinuity is three dimensional (3D).

Scattering problems in waveguiding structures can be formulated in terms of matching a set of higher order modes across some interface [1,2] or at a set of points on a boundary [3,4]. Application of the method of moments yields a set of integral equations [5] which typically require knowledge of a Green's function for the waveguide [6,7]. Christ and Hartnagel [8] employ a finite-difference method to obtain a set of linear equations, solved simultaneously to obtain the scattering parameters of arbitrary obstacles without use of Green's function. Hayata, et. al [9] have used the finite element method to calculate the actual fields in lossy samples. These methods require significant computation time.

In this paper we approach the 3D scattering of electromagnetic waves by expanding the scattered fields in terms of the empty stripline eigenmodes which have been calculated numerically using the finite element method [10,11]. These basis modes are then coupled in the volume of the material discontinuity by an application of the Reciprocity Theorem.

DESCRIPTION OF THE METHOD

Consider the stripline shown in Fig.1. A material body inhomogeneously loads the transmission line.Its constitutive parameters are complex: $\mu=\mu'-j\mu''$, $\epsilon=\epsilon'-j\epsilon''$. The fields in the transmission line are represented as a superposition of the empty guide modes. These are obtained as follows:

(i) The TEM mode:

$$E = A\nabla_t\Phi(x,y)\exp(-j\beta_0 z)$$

$$\text{where:}\ \beta_0=\omega(\mu_0\epsilon_0)^{1/2}$$

and Φ is the solution of the 2D Laplace equation subject to Dirichlet boundary conditions.

(ii) Higher Order modes: E and H modes are obtained from scalar potentials Φ_t which satisfy the 2D Helmholtz equation of the form:

$$\nabla^2\Phi_t(x,y)+k_t^2\Phi_t=0$$

Here k_t is the cutoff wave number for the t^{th} mode. The boundary conditions that apply on the perfectly conducting boundaries of the transmission line are: E modes: $\Phi=0$, H modes: $\partial\Phi/\partial n=0$ In this work the empty guide eigenmodes were computed by means of the Finite Element Method [1],[2]. The eigenmode potential distributions are stored in computer files. Next the scattered fields are determined by an application of the Reaction Method [12].

REACTION METHOD CALCULATION OF THE SCATTERED FIELDS

The first step consists in representing the discontinuity by means of a T network with elements as shown in Fig.2. The network is reciprocal since the sample is assumed isotropic. A solution of the discontinuity problem consists in determining the values of Z_a, Z_b

. To solve for the two-port parameters the network is decomposed into two one-ports having each a single impedance element as follows. Let z=0 be the sample's mid-plane. Now let an even excitation be applied; i.e. the network is subjected to two equal incident waves from both z<0 and z>0 phased so that at z=0 a maximum of the tangential electric field occurs while the tangential magnetic field vanishes. From symmetry the scattered tangential magnetic field will also be zero at z=0. Hence a magnetic wall can be placed over z=0 without affecting the field (see Fig.2b).

For an odd excitation two equal incident waves are phased so that at z=0, $E_t=0$ whereas H_t- Maximum. From symmetry the scattered tangential electric field will also be zero at z=0. Hence an electric wall can be placed here without affecting the field (see Fig.2c). Thus the problem is reduced to solving for two one-ports. For simplicity the Reaction Method will be described as it applies to the one-port shown in Fig.2c.

The stripline can be separated into two sections. The first is the empty portion,Region I in Fig.1. The second is the loaded section.

In Region I, z <= -1

$$E = e_0[\exp(-j\beta_0 z)+\exp(j\beta_0 z)] + \Sigma R_n e_n \exp(jk_n z) \quad (1a)$$

$$H = Y_0(z\times e_0)[\exp(-j\beta_0 z)+\exp(+j\beta_0 z)] - \Sigma R_n Y_n(z\times e_n)\exp(jk_n z) \quad (1b)$$

Here β_0 is the propagation constant for the TEM mode and $k_n=\beta_n-j\alpha_n$ is the complex propagation constant for the n^{th} of the empty guide. The quantities e_0 and e_n are the transverse field distributions for the respective modes. In what follows subscript zero (n=0) refers to the TEM mode. The Y_n are the modal admittances and R_n is the complex electric field amplitude of the scattered waves.

In Region II, -l <= z <= 0, the fields are:

$$E=\Sigma B_n e_n(x,y)[\exp(-j\Gamma z)-\exp(+j\Gamma z)] \quad (2a)$$

$$H=\Sigma C_n e_n(x,y)[\exp(-j\Gamma z)+\exp(+j\Gamma z)] \quad (2b)$$

Notice that we have assumed that only the fundamental loaded guide mode is excited,that it propagates with a constant Γ and that it is made up of a number (N+1) of the empty guide modes. Thus one has 3(N+1) unknowns, R_n, B_n, C_n (n=0,...N), and this many equations are needed. It should also be noted that Γ is not known. The additional equations are obtained as follows.

At z=-l, the transverse fields must be continuous. Hence, equating Eqs(1) and (2) yields:

$$\left[\exp(+j\beta_0 l)-\exp(-j\beta_0 l)\right]e_0+\Sigma R_n e_n \exp(-jk_n l)$$

$$=\Sigma B_n e_n(x,y)[\exp(+j\Gamma l)-\exp(-j\Gamma l)] \qquad (3a)$$

$$Y_0(z\times e_0)\left[\exp(+j\beta l)+\exp(-j\beta_0 l)\right]$$

$$-\Sigma R_n Y_n(z\times e_n)\exp(-jk_n l)$$

$$=\Sigma C_n e_n(x,y)[\exp(+j\Gamma l)-\exp(-j\Gamma l)] \qquad (3b)$$

Multiplying these equations by the modal field functions e_n ,integrating over the guide's cross section and using mode orthogonality,lead to 2(N+1) linear equations. The additional (N+1) equations needed to solve the system are obtained from application of the Reciprocity Theorem,which relates the incident and scattered fields and their excitations. It can be stated as:

$$\int\int\left(E^i\times H^s-E^s\times H^i\right)\cdot n\,da$$

$$=-\int\int\int E^i\cdot J-H^i\cdot K\,dV \qquad (4)$$

where: $J=j\omega(\epsilon-\epsilon_0)E$, $K=j\omega(\mu-\mu_0)H$.

Here the excitations J and K are the electric and magnetic volume polarisation currents induced by the total fields.

Applying the Reciprocity Theorem to the normal modes e_n through the use of Eqs (1) and (2) yields:

$$-2R_n=-j\omega(\epsilon-\epsilon_0)\int\int\int \Sigma B_n e_n(x,y)\exp(-j\Gamma z)$$

$$-\exp(+j\Gamma z)E^i dV$$

$$+j\omega(\mu-\mu_0)\int\int\int \Sigma C_n e_n(x,y)\exp(-j\Gamma z)$$

$$+\exp(+j\Gamma z)H^i dV \qquad (n=0,\ldots\ldots N)(5)$$

which is another set of (N+1) equations. Solution of the linear equation system permits the evaluation of the impedance element of Fig.2c by taking the ratio between the electric and magnetic fields at z=-l, if Γ is known.

PROPAGATION CONSTANT CALCULATION

Let the total electric field in the loaded region be given by:

$$E^{\pm}=E^{\pm}(x,y)\exp(\mp jkz)$$

$$=(E_t\pm zE_z)\exp(\mp jkz) \qquad (6)$$

Then from Maxwell's Equations it follows that [13]:

$$\Gamma^2\int\int\mu^{-1}E_t^2 da-2\Gamma\int\int\mu^{-1}E_t\cdot\nabla E_z da+$$

$$\int\int\mu^{-1}(\nabla\times E^+)\cdot(\nabla\times E^-)-\omega^2\epsilon E^+\cdot E^- da=0 \qquad (7)$$

This equation yields a stationary value for Γ if $n x E=0$ on the boundary. This condition is satisfied here. If the field is assumed to be nearly TEM then:

$$E_z=0,\nabla\times E^+=\nabla\times E^-=0,$$

leading to: $$\Gamma^2=\frac{\omega^2\int\int\epsilon E_t^2 da}{\int\int\mu^{-1}E_t^2 da} \qquad (8)$$

RESULTS

Results presented here were computed assuming that the field distribution in the loaded region consisted of the TEM mode only. Work on higher order modes solution is under way. The theory was tested by making calculations for the fully filled case,and for shielded microstrip. Comparison with the equations of Edwards [14] for shielded microstrip with a thick center conductor gave 1% agreement.

Figure 3 shows the predicted loaded region propagation constant, Γ ,as a function of frequency for a loss free dielectric which partially fills the stripline as indicated in Fig.1. The stripline geometry (in dimensionless units) is given by W=500, S=100, 2h+t=200, t=10. The sample's dimensions are h=100, A=40, L=2.5 (cm). Also indicated on the Figure is the propagation constant of the empty structure. Notice that the discontinuity fills only about 4% of the cross sectional area. Figure 4 shows results for some lossy cases with the same dimensions. Running time for the S-parameters on an IBM AT is approximately 2 sec. for each frequency.

CONCLUSIONS

The agreement found between the computed results and the test problems is very encouraging. Once the higher order modes are added both the accuracy of the solutions as well as the possible range of values for the material parameters will be considerably enhanced. We believe that the approach is effective for the solution of 3D scattering problems in waveguides as well as for the computing of discontinuities.

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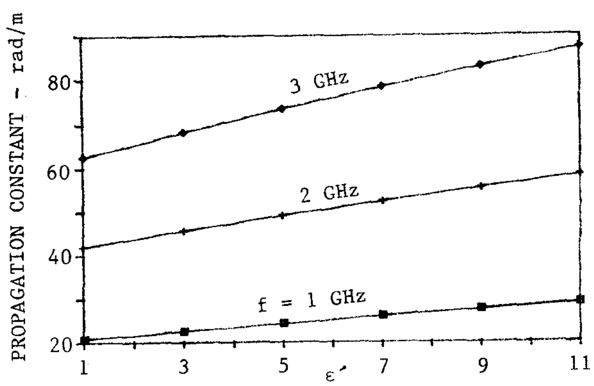


FIG. 3. PROPAGATION CONSTANT OF PARTIALLY FILLED LOSS FREE LINE AS A FUNCTION OF ε'

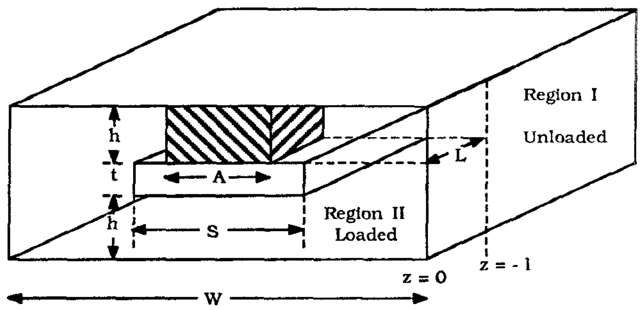


FIG. 1. SAMPLE LOADED STRIPLINE

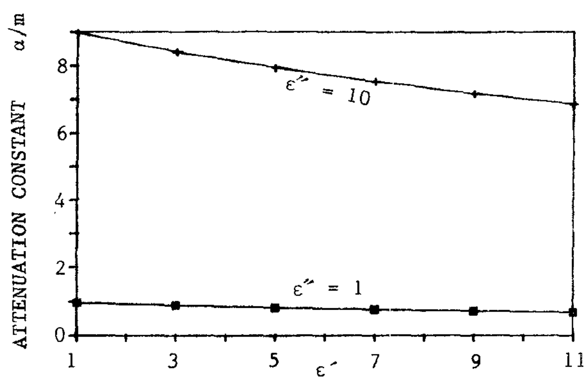


FIG. 4a.

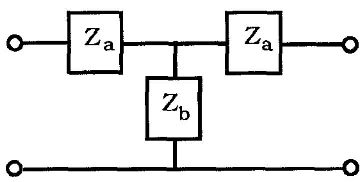
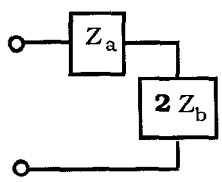


Fig. 2a Complete Equivalent Circuit



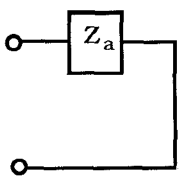


Fig. 2b
Even Mode Excitation
Magnetic Wall at mid-plane

Fig. 2c
Odd Mode Excitation
Electric Wall at mid-plane

Fig.2 Equivalent Circuit Representation for Symmetric Obstacle

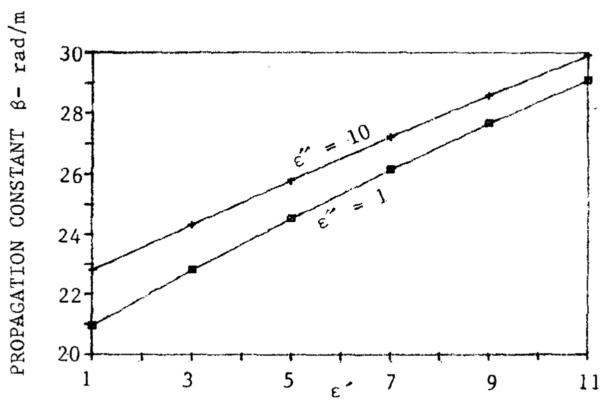


FIG. 4b.

FIG. 4. COMPLEX Γ AS FUNCTION OF ε'

a.) α vs ε'

b.) β vs ε'